

```
## Loading required package: ggplot2
## rstan (Version 2.9.0, packaged: 2016-01-05 16:17:47 UTC, GitRev:
05c3d0058b6a)
## For execution on a local, multicore CPU with excess RAM we recommend
calling
## rstan_options(auto_write = TRUE)
## options(mc.cores = parallel::detectCores())
```

STAT 544 Mid-term Exam  
Thursday 10 March 8:00-9:20

Instructor: Jarad Niemi

**INSTRUCTIONS**

Please check to make sure you have 5 pages with writing on the front and back (some pages are marked 'intentionally left blank'). Feel free to remove the last page, i.e. the one with R code.

On the following pages you will find short answer questions related to the topics we covered in class for a total of 50 points. Please read the directions carefully.

You are allowed to use a calculator and one  $8\frac{1}{2} \times 11$  sheet of paper with writing on both front and back. A non-exhaustive list of items you are not allowed to use are **cell phones, laptops, PDAs, and textbooks**. Cheating will not be tolerated. Anyone caught cheating will receive an automatic F on the exam. In addition the incident will be reported, and dealt with according to University's Academic Dishonesty regulations. Please refrain from talking to your peers, exchanging papers, writing utensils or other objects, or walking around the room. All of these activities can be considered cheating. **If you have any questions, please raise your hand.**

You will be given only the time allotted for the course; no extra time will be given.

Good Luck!

Please print your name below:

Student Name: \_\_\_\_\_

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## Diagnostic testing

1. Details of two diagnostic tests for a fairly prevalent disease is given below.

Test	Sensitivity	Specificity
CDC	0.90	0.80
MD	0.70	0.60

Recall that sensitivity is the probability of a positive test result when the individual has the disease and specificity is the probability of a negative test result when the individual does not have the disease. Suppose the prevalence of this disease is one case per ten individuals.

- (a) Determine the marginal probability of a positive CDC test. Show all your work and define all your notation. (5 pts)

- (b) Calculate the probability of having Zika for an individual with a positive CDC test. (5 pts)

The table below provides the cost for each combination of true disease state and diagnostic test result.

Truth	Test result	Cost (\$)	Probability	
			CDC	MD
Disease	Positive	0		
Disease	Negative	10		
No disease	Positive	1		
No disease	Negative	0		

- (c) Complete the table above by calculating the probabilities for each truth-test result combination for each of the two diagnostic tests. Define your notation and show your work here. (5 pts)

- (d) The cost for the CDC test is \$50 while the cost for the MD test is \$10. Determine the optimal diagnostic test to use. (5 pts)

## Prior-posterior

2. Suppose  $Y$  has the probability density function

$$p(y|\theta) = (\theta + 1)y^\theta \mathbf{I}(0 < y < 1) \text{ for } \theta > 0.$$

- (a) Derive Jeffreys prior for this model. (If you are having trouble, I will give you the answer and mark off 2 points. You can still obtain the remaining 6 points by deriving the answer now that you know it.) (8 pts)

- (b) Is Jeffreys prior proper? (2 pt)

(c) Derive the posterior for one observation under Jeffreys prior. (4 pts)

(d) When is this posterior proper? (Hint:  $y^\theta = e^{\theta \log y}$ .) (3 pts)

(e) Find a 95% HPD interval for  $\theta$  for a single observation  $y$ . (3 pts)

## Pilot training

3. Airplane pilots were randomly assigned to a group where each group used a different teaching method. The pilots in the group were randomly assigned to a training scenario. The result of an individual pilot's success on the training scenario was binary: a 1 if the pilot successfully met the scenario's objectives and a 0 otherwise. Each group-scenario combination had from 5 to 8 pilots and the response  $p$  is the proportion of pilots in that group-scenario combination to successfully satisfy the scenario's objectives.

(a) Using statistical notation, write down the model (including priors) encoded by Stan. (5 pts)

(b) Are the priors on the standard deviations reasonable? Why or why not? (3 pts)

(c) Are you concerned about convergence of **this** Markov chain? Why or why not? (2 pts)

- (d) Provide a 95% credible interval for the overall proportion of successes. (1 pt)
- (e) Are the mean proportion of successes for groups or scenarios more variable? Explain your answer. (2 pts)
- (f) Suggest a test statistic for which you could calculate a posterior predictive pvalue that could identify heavy-tails in the scenario means and defend this suggestion. (3 pts)
- (g) Using statistical notation, write down a model (without priors) that accommodates the discreteness of the response. (4 pts)

## Stan code

```
pilots_stan = "  
data {  
  int<lower=0> N;  
  int<lower=0> n_groups;  
  int<lower=0> n_scenarios;  
  int<lower=1,upper=n_groups> group_id[N];  
  int<lower=1,upper=n_scenarios> scenario_id[N];  
  vector[N] p;  
}  
parameters {  
  vector[n_groups] gamma;  
  vector[n_scenarios] delta;  
  real eta;  
  real<lower=0,upper=1> sigma_gamma;  
  real<lower=0,upper=1> sigma_delta;  
  real<lower=0,upper=1> sigma;  
}  
transformed parameters {  
  vector[N] mu;  
  
  for (i in 1:N)  
    mu[i] <- eta + gamma[group_id[i]] + delta[scenario_id[i]];  
}  
model {  
  gamma ~ normal(0, sigma_gamma);  
  delta ~ normal(0, sigma_delta);  
  p ~ normal(mu, sigma);  
}  
"  
library(rstan)  
m = stan_model(model_code = pilots_stan)  
r = sampling(m,  
  data = dat,  
  pars = c('mu'), include = FALSE, # removes mu from the output  
  iter = 20000,  
  chains = 4)
```

## Stan results

r

```
## Inference for Stan model: a5814b234db6af24770e1cd8dc27fa6d.
## 4 chains, each with iter=20000; warmup=10000; thin=1;
## post-warmup draws per chain=10000, total post-warmup draws=40000.
##
##           mean se_mean  sd  2.5%  25%  50%  75% 97.5% n_eff Rhat
## gamma[1] -0.01  0.00 0.06 -0.15 -0.04 -0.01  0.01  0.10 6357  1
## gamma[2] -0.01  0.00 0.06 -0.14 -0.03  0.00  0.02  0.11 6023  1
## gamma[3]  0.00  0.00 0.06 -0.12 -0.02  0.00  0.03  0.13 5959  1
## gamma[4] -0.01  0.00 0.06 -0.15 -0.03  0.00  0.01  0.10 6434  1
## gamma[5]  0.03  0.00 0.06 -0.08 -0.01  0.01  0.05  0.18 5039  1
## delta[1] -0.09  0.00 0.17 -0.45 -0.20 -0.09  0.02  0.25 3409  1
## delta[2] -0.28  0.00 0.18 -0.64 -0.39 -0.28 -0.17  0.06 3387  1
## delta[3]  0.00  0.00 0.18 -0.35 -0.11  0.00  0.11  0.34 2762  1
## delta[4] -0.22  0.00 0.17 -0.58 -0.33 -0.22 -0.12  0.12 4312  1
## delta[5] -0.02  0.00 0.17 -0.37 -0.12 -0.02  0.09  0.33 3342  1
## delta[6]  0.49  0.00 0.18  0.15  0.37  0.48  0.60  0.84 4012  1
## delta[7] -0.31  0.00 0.18 -0.67 -0.41 -0.30 -0.20  0.03 4222  1
## delta[8]  0.44  0.00 0.18  0.10  0.33  0.44  0.55  0.80 3467  1
## eta      0.44  0.00 0.16  0.12  0.35  0.44  0.53  0.76 2831  1
## sigma_gamma 0.07  0.00 0.07  0.01  0.02  0.05  0.08  0.25 3048  1
## sigma_delta 0.39  0.00 0.14  0.21  0.30  0.37  0.46  0.74 7796  1
## sigma      0.23  0.00 0.03  0.18  0.21  0.22  0.25  0.30 10118  1
## lp__      51.60  0.11 4.97 41.38 48.43 51.69 54.99 61.06 2127  1
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 10 09:51:40 2016.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```