

Name \_\_\_\_\_

Spring 2018

STAT 401C

Exam I  
(100 points)

**Instructions:**

- Full credit will be given only if you show your work.
- The questions are not necessarily ordered from easiest to hardest.
- You are allowed to use any resource except aid from another individual.
- Aid from another individual, will automatically earn you a 0.

1. A diagnostic test for disease D has a sensitivity of 0.95 and a specificity of 0.9. The prevalence of the disease is 0.02. (20 points)
- (a) Define notation for the following events (1 point each).
- having the disease
  - not having the disease
  - testing positive
  - testing negative
- (b) Use the notation in the previous step to define the following probabilities (2 points each).
- sensitivity
  - specificity
  - prevalence
- (c) If an individual tests positive for the disease, what is the probability they actually have the disease? (10 points)

2. Let  $X$  be a random variable with the following probability mass function:

$x$	-10	-5	0	5	10
$P(X=x)$	0.3	0.2	0.1	0.2	0.2

(a) Is  $P(X = x)$  a valid probability mass function? Why or why not? (5 points)

(b) Calculate  $E[X]$ . (5 points)

(c) Let  $Y = |X|$  what is the probability mass function for  $Y$ ? (5 points)

(d) Find  $E[|X|]$ . (5 points)

3. Answer the following questions based on this joint distribution for the random variables  $X$  and  $Y$ .

X	Y		
	1	2	3
-1	0.1	0.2	0.1
0	0.1	0.1	0.1
1	0.1	0.1	0.1

- (a) What is the image for the random variable  $Y$ ? (2 points)
- (b) Find the marginal probability mass function for  $X$ . (6 points)
- (c) Find  $P(Y > X)$ . (6 points)
- (d) Are  $X$  and  $Y$  independent? Why or why not? (6 points)



5. A positive displacement pump is used to fill an ethanol tanker. The pump pumps 1 gallon of ethanol at a time with a mean of 1 gallon and a standard deviation of 0.01 gallons and independently of all other measurements. Assume each pump is normally distributed.

(a) If the pump, pumps 30,001 times, what is the probability that the true amount of ethanol in the tanker is less than 30,000 gallons? (10 points)

(b) How many pumps should the pump pump to ensure that the true amount is greater than 30,000 gallons with probability 99%. (10 points)