

Name _____

Fall 2021

STAT 587-1

Exam I
(70 points)

Instructions:

- Write your name on the top, but do not open the exam.
- You are allowed to use any resource except aid from another individual.
- A total of 4 pages with a front and back.
- For full/partial credit, show all your work.

1. The United States has identified a previously unknown piece of space debris and is trying to determine the probability it is from a US space craft.

For identified space debris, we have the following proportions by country

##	Russia	US	China	Other
##	0.4	0.3	0.2	0.1

The space debris is giving off a signature in the infrared spectrum. The following table provides the proportion of existing space debris for each country that has this particular signature, i.e. the conditional probability given the country:

##	Russia	US	China	Other
##	0.01	0.10	0.05	0.02

Given that the space debris is giving off this signature, determine the probability that the debris belongs to Russia, US, China, and Other. (10 points)

2. For the following variables, identify the most appropriate distribution to model the variable from this list: binomial, Poisson, uniform, and normal. (1 point each)
- (a) strength of #3 carbon steel rebar
 - (b) weight of dust in my office
 - (c) number of occupied chairs in Gilman 2272 during this exam (there are a total of 70 chairs)
 - (d) number of car accidents in Story County in August 2021
 - (e) number of visitors to a website who click on an advertisement out of the next one thousand visitors
 - (f) elasticity of a polymer network
 - (g) time it will take to read *The Black Swan*
 - (h) number of student organizations at Iowa State University
 - (i) number of books on my office bookshelf, out of the 74 books on my bookshelf, with the word "Statistics" in the title
 - (j) thickness of my office window

3. As a road is constructed, there is an average of 1.5 defects per mile segment of the road. Let Y_i be the number of defects on segment i . Assume $Y_i \sim Po(1.5)$ and that the number of defects in each segment is independent.

(a) The following questions are about the first road segment.

- What is the expected number of defects in the first segment? (2 points)

- What is the **standard deviation** of the number of defects in the first segment? (2 points)

- Calculate the probability of no defects in the first segment. (3 points)

- Calculate the probability of two or more defects in the second segment. (3 points)

(b) The following questions are about a 10 mile stretch of road. Assume the number of defects in each mile stretch is independent. (3 points each)

- What is the expected number of total defects in a 10 mile stretch of the road? (2 points)

- What is the **standard deviation** in the number of total defects in a 10 mile stretch of the road? (2 points)

- What is the probability of exactly 14 defects in a 10 mile stretch of the road? (3 points)

- What is the probability of 12 or fewer defects in a 10 mile stretch of the road? (3 points)

4. Assume the compressive strength of residential concrete follows a normal distribution with a mean of 17 MPa and a standard deviation of 2 MPa. (2 points each)
- What is the expected compressive strength?
 - What is the **variance** on the compressive strength?
 - The coefficient of variation (*cv*) is the standard deviation divided by the mean. What is the coefficient of variation for compressive strength of residential concrete?
 - What is the probability the compressive strength of a particular sample of residential concrete is below 15 MPa?
 - Provide an interval such that 95% of all residential concrete samples will have compressive strength within that interval.

A company has indicated that they can double the compressive strength plus 5 Mpa by adding a proprietary ingredient. Specifically, if Y is the normal compressive strength, the company can make it $2Y + 5$.

(f) What is the expected compressive strength of the concrete with this ingredient? (3 points)

(g) What is the **variance** in compressive strength of the concrete with this ingredient? (3 points)

(h) What is the probability the new strength will be greater than 40? (4 points)

5. An extruding machine has a screw that propels plastic through a barrel to be extruded. Each turn of the screw pushes, on average, a 5 millimeter (mm) length of plastic with a standard deviation of 0.3 mm.

(a) After 70 turns of the screw, what is the expected length of the extruded plastic? (2 points)

(b) After 70 turns of the screw, what is the **standard deviation** of the length of the extruded plastic? (2 points)

(c) What is the probability the extruded plastic after 70 turns is less than 355 mm in length? (3 points)

(d) The machine must be adjusted to ensure that after 70 turns the probability the extruded plastic is less than 355 mm is, at most, 0.99. What is the maximum standard deviation for a single turn required to ensure this probability? (3 points)