Name ____

Fall 2021 **STAT 587-2** **Exam II**

(60 points)

Instructions:

- You are allowed to use any resource except aid from another individual.
- Aid from another individual, will automatically earn you a 0.

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1. Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$ and we construct an equal-tail $100(1-a)\%$ credible interval for μ. We are interested in the width of the credible interval and how it changes in response to changing the following. For each proposed change (assuming everything else stays the same), indicate whether the credible interval gets wider, narrower, stays the same, or cannot determine based on the available information. (2 pts each)

   (a) a gets larger

   (b) sample mean gets larger

   (c) sample variance gets larger

   (d) sample size increases

   (e) units of Y_i are changed from kilometers to meters

2. Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$. The following are two-sided 95% confidence intervals for μ. What can be said about the p-value for the test $H_0 : \mu = 0$ vs $H_A : \mu \neq 0$? (2 pts each)

   (a) (-1,10)

   (b) (1,2)

   (c) (0,2)

3. Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$. The following are p-values for the test $H_0 : \mu = 0$ vs $H_A : \mu \neq 0$. What can be said about a two-sided 95% confidence interval for μ? (2 pts each)

   (a) $p = 0.01$

   (b) $p = 0.1$

4. To detect injury in pigs, researchers conduct a study using a random set of already injured pigs. For each pig, the researcher uses a scale under each foot while a pig is eating. The force from each foot is measured and the researcher calculates the difference (injured foot minus non-injured foot). The researcher hypothesizes that there will be less weight on the injured foot as the pig compensates for the injury.

   Let Y_i be the difference for pig i and assume $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$. The following statistics are observed

n	sample mean	sample standard deviation
10	0.8	1.5

   (a) Provide an interpretation for μ, i.e. what is μ (in words)? (2 pts)

   (b) What is $\hat{\mu}_{MLE}$? (2 pts)

   (c) Compute a two-sided 80% confidence interval for μ. (6 pts)

(d) Compute a p-value for testing $H_0 : \mu = 0$ vs $H_A : \mu \neq 0$. (6 pts)

(e) State a conclusion based on the calculated p-value in the previous question? (2 pts)

(f) What is a more appropriate alternative hypothesis based on the researchers hypothesis? (2 pts)

5. A manufacturer produces nylon with an intended modulus of elasticity of 2.7 GPa. A random sample of nylon from the factory results in 3 of 100 samples with a modulus of elasticity less than 2.7 GPa. Let Y be the number of samples less than 2.7 GPa and assume $Y \sim Bin(n, \theta)$ where θ is the population proportion of samples that have modulus of elasticity less than 2.7 GPa.

   (a) Determine the Bayes estimator for θ, i.e. the posterior expectation. (2 pts)

   (b) Determine a 70% equal-tail credible interval. (2 pts)

   (c) Determine the probability that θ is less than 0.05. (2 pts)

   (d) Draw a graph of the posterior for θ. (4 pts)

6. The file `data.csv` contains data that are assumed to be normally distributed. Answer the following questions based on these data.

   (a) How many observations are there? (2 pts)

   (b) What is the sample mean of the data? (2 pts)

   (c) What is the sample variance of the data? (2 pts)

   (d) Construct a two-sided 98% confidence interval for the population mean. (2 pts)

   (e) Compute a p-value for the test with alternative hypothesis that the population mean is greater than 34. (2 pts)